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MODEL OF THE FLEXIBLE SYNCHRONOUS PRODUCTIVE LINE WITH REDUNDANT TECHNOLOGICAL CELL

Abstract

This paper introduces the mathematical model of operating the flexible synchronous productive line (FSPL) of multifunctional CNC machines, that includes one redundant multifunctional CNC machine which can take over functions of every FSPL machine. The graph of FSPL state, relations and equations used to calculate the reliability and the productivity were shown. Maple – the software used for reliability and productivity calculations and modelling as well as the mathematical results were presented.

1. INTRODUCTION

Multirole CNC machines are mainly designed for processing frame type parts, which have many holes with different diameters, precision (class 5 to 11), on which resistance points are based, additional tools are connected to the frame and to mounting connections (to attach the part using screws or pegs, making processing, basing and assembly easier). The dimensions of the main holes diameters hesitate in the wide range (16 to 50 mm) and depend on the type of part [1]. The work [2] introduces specification of processing and classification of holes for the system of automated designing of technological processes. Modern market conditions require production quick starting and quick changing of the assortment of produced parts. CNC machine and Flexible Productive Systems (FPS), uniting the high flexibility of traditional equipment and the high efficiency of machine, are the most effective equipment for the multinomenclature production.

2. THE METHODOLOGY OF MODELING THE FSPL RELIABILITY AND EFFICIENCY

Every multirole CNC machine can be considered as a complex system. If the system contains „n” of serial connected elements, damage of any of them leads to the failure of whole system, and can be described by graph [3, 4].

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States on the graph: S_0 – all n elements of the system are operating; S_1 – the first element failed and the system is nonoperational; S_2 – the second element failed and the system is out of order; \dots ; S_n – n^{th} element failed and the system is not working.

Indications on the graph:

$\lambda_i, (i = 1, \bar{n}); \mu_i, (i = 1, \bar{n})$ – the intensity of failure and restoration stream of working ability of 1 to n elements.

Because of the fact that after failure of any elements rest of the elements can't function properly on the time of restoration of his work ability, it is considered that only one element can simultaneously fail. All failure and restoration streams are considered as simple.

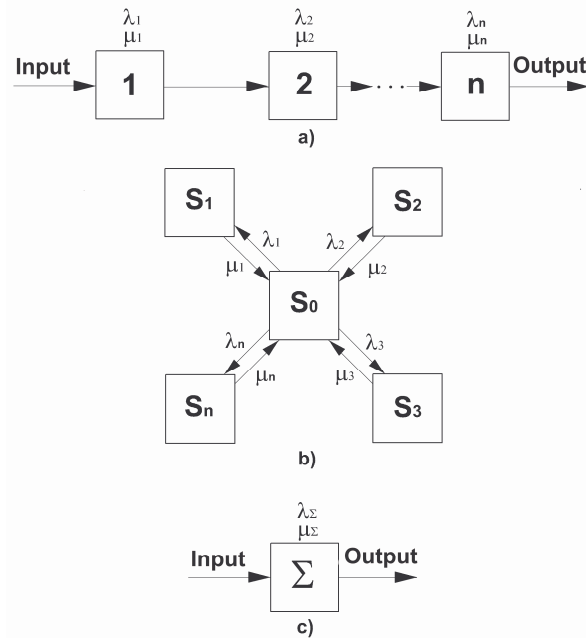


Fig.1. The conditions graph of the multirole CNC machine: a) system elements from 1 to n ; b) elements conditions; c) machine as the sum of all elements

The system of equations for the qualification of final probabilities is presented below:

$$\left\{ \begin{array}{l} P_0 \sum_{i=1}^n \lambda_i = \sum_{i=1}^n P_i \mu_i; \\ P_1 \mu_1 = P_0 \lambda_1; \\ P_2 \mu_2 = P_0 \lambda_2; \\ \dots\dots\dots \\ P_i \mu_i = P_0 \lambda_i; \\ \dots\dots\dots \\ P_n \mu_n = P_0 \lambda_n. \end{array} \right. \quad (1)$$

The standardization condition:

$$\sum_{j=0}^n P_j = 1. \quad (2)$$

After change of the first equation of system (1) to the standardization condition (2) and solutions, every probability $P_i, (i = \overline{1, n})$ is expressed by P_0 :

$$P_i = P_0 \frac{\lambda_i}{\mu_i}. \quad (3)$$

Set of numbers i is marked as $I (i \in I)$. Let's introduce the j , belonging to this set: $j \in I$. With the regard of new letters, after the substitution 3 to the standardization condition 2 following formula is received:

$$P_0 = \frac{1}{1 + \sum_{j=1}^n \frac{\lambda_j}{\mu_j}}. \quad (4)$$

After substitution (4) to (3):

$$P_i = \frac{\lambda_i}{(1 + \sum_{j=1}^n \frac{\lambda_j}{\mu_j}) \mu_i} = \frac{\rho_i}{1 + \sum_{j=1}^n \rho_j}, \quad (5)$$

where: $\rho_i = \frac{\lambda_i}{\mu_i}, \rho_j = \frac{\lambda_j}{\mu_j}$.

The output system (fig.1) is replaced with the simple two states element: working and non-work (in the damage condition; nonoperational). Graph of such element or new system is introduced on fig. 2.

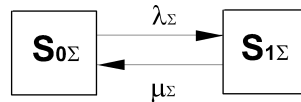


Fig.2. The graph of the system element

λ_{Σ} is defined as:

$$\lambda_{\Sigma} = \sum_{i=1}^n \lambda_i. \quad (6)$$

The value μ_{Σ} is defined from dependence:

$$\mu_{\Sigma} = \frac{P_0}{1 - P_0} \lambda_{\Sigma} \quad (7)$$

After substitution (4) to (7) following formula is received:

$$\mu_{\Sigma} = \frac{\lambda_{\Sigma}}{\sum_{j=1}^n \rho_j}. \quad (8)$$

3. THE MATHEMATICAL MODEL OF FSPL FUNCTIONING

At present we use the structure of the flexible synchronous productive line (FSPL) from the multirole CNC machines with redundant technological cell (RTC) which can replace any multirole machine – technological cell (TC). Fig.3 introduces the structure of such flexible system (FSPL).

The redundant technological cell (RTC) can replace only one damaged machine (TC), so whole system (FSPL) stops to work after failure of two machines (TC) [5].

The graph of states (FSPL), including RTC, is introduced on fig. 4. States on the graph:

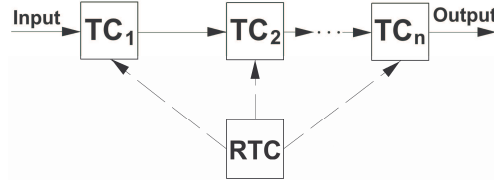


Fig.3. The FSPL structure

S_0 - all multirole machines (TC) are operating; S_1 - 1st TC₁ don't operating; S_2 - 2nd TC₂ don't operating; ... , S_n - n - e TC _{\bar{n}} don't operating; $S_{1,1}$ - second TC₂ failure while TC₁ don't operating; .. $S_{1,2}$ - third TC₃ failure while TC₁ don't operating; ...; $S_{1,n-1}$ - the n - e TC _{\bar{n}} failure while TC₁ don't operating; $S_{2,1}$ - first TC₁ failure while TC₂ don't operating; $S_{2,2}$ - third TC₃ failure while TC₂ don't operating; ...; $S_{2,n-1}$ the n - e TC _{\bar{n}} failure while TC₂ don't operating; $S_{3,1}$ - first TC₁ failure while TC₃ don't operating; $S_{3,2}$ - second TC₂ failure while TC₃ don't operating; $S_{3,3}$ - (not shown on graph) the TC₄ failure while TC₃ don't operating; ... ; $S_{3,n-1}$ - the n -e TC _{\bar{n}} failure while TC₃ don't operating; $S_{i,1}$ - first TC₁ failure while TC _{\bar{i}} don't operating; $S_{i,2}$ - second TC₂ failure while TC _{\bar{i}} don't operating; ... ; $S_{i,n-1}$ - failure n -e TC _{\bar{n}} while TC _{\bar{i}} don't operating; $S_{n,1}$ - first TC₁ failure while TC _{\bar{n}} don't operating; $S_{n,2}$ - second TC₂ failure while TC _{\bar{n}} don't operating; ... ; $S_{n,n-1}$ - $n-1$ - e TC _{\bar{n}} failure while TC _{\bar{n}} don't operating; States $S_0, S_1, S_2, S_3, S_4, \dots, S_n$ - able to the work and remaining emergency. Graph clarification: $\lambda_i (i = 1, \bar{n}); \mu_i (i = 1, \bar{n})$ - the intensity of the failure stream and restoring the working ability of technological devices TC _{\bar{i}} $i = 1, \bar{n}$.

The number of states is considerable (eg. for $n = 10$ the number of states is $N = n^2 + 1 = 101$, that makes construction and analysis of model difficult). That is why approach leaning on the increase of states is proposed.

We isolate following subsets in E set (power N):

$$E_1 = \{S_1, S_{1,1}, S_{1,2}, \dots, S_{1,n-1}\}; E_2 = \{S_2, S_{2,1}, S_{2,2}, \dots, S_{2,n-1}\}; E_3 = \{S_3, S_{3,1}, S_{3,2}, \dots, S_{3,n-1}\};$$

$$\dots; E_i = \{S_i, S_{i,1}, S_{i,2}, \dots, S_{i,n-1}\}; \dots; E_n = \{S_n, S_{n,1}, S_{n,2}, \dots, S_{n,n-1}\}.$$

We will qualify the probability of system existance in these subsets. In this case we will consider graph of equivalent enlarged system shown by fig. 5.

States on graph (fig.5):

S_0 - all multirole CNC machines are operating; $S_{1\Sigma}$ - the system is in one of states of the E_1 subset; $S_{2\Sigma}$ - the system is in one of states of the E_2 subset; ... ; $S_{n\Sigma}$ - the system is in one of states of the E_n subset.

On the graph: $\lambda_i, (i = 1, \bar{n})$ - is the intensity of the failure streams $UT_i, (i = 1, \bar{n})$; $\mu_{i0}, (i = 1, \bar{n})$ - the intensity of the stream restoring the system working ability from subsets $E_i, (i = 1, \bar{n})$.

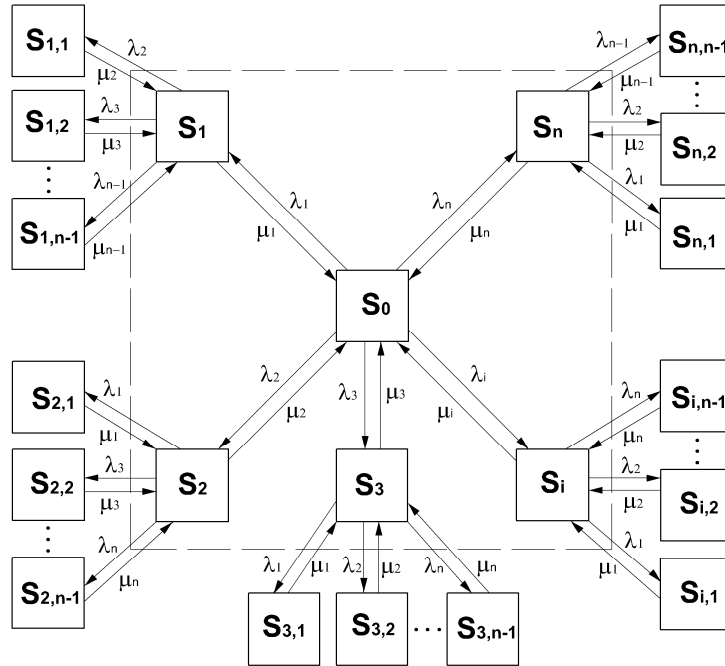


Fig. 4. Graph of FSPL conditions, including one reserved RTC place

The task consists in the defining $\mu_{i0} (i = 1, \bar{n})$. If all graph (fig. 4) conditions probabilities were known, then $\mu_{i0} (i = 1, \bar{n})$ can be determined from dependence:

$$\mu_{i0} = \frac{P_i}{P_i + \sum_{j=1(j \neq i)}^{n-1} P_{ij}} \mu_i \quad (9)$$

where P_{ij} - the states probability of $S_{ij} \in E_i$, the rate before μ_i in (1), equal $\frac{P_i}{P_i + \sum_{j=1(j \neq i)}^{n-1} P_{ij}}$,

then conditional probability, that it is being in the subset of states E_i ,

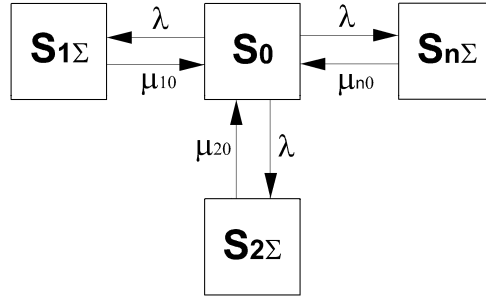


Fig. 5. Graph of conditions equivalent of enlarged system

the system is in the state S_j .

We will mark the component of E set as $S_k, (k = \overline{1, N}), (S_k \in E)$. Dividing numerator and nominative (1) by the probability of system being in the E_i $P_{iz} = P\{S_k \in E_i\} = P_i + \sum_{j=1(j \neq i)}^{n-1} P_{ij}$ subset, we will receive:

$$\mu_{i0} = \frac{P_i}{P_{i\Sigma}} \mu_i = P_{iy} \mu_i \quad (10)$$

where P_{iy} - the conditional probability of system being in the S_i state.

We will qualify the conditional probability of the elements of E_i subsets:

$P_{iy} = P\{S_k = S_i / S_k \in E_i\}; P_{ijy} = P\{S_k = S_i / S_k \in E_i\}$. They are equal:

$$P_{iy} = \frac{P_i}{P_{i\Sigma}}, \quad (11)$$

$$P_{ijy} = \frac{P_{ij}}{P_{i\Sigma}} = \frac{P_{ij}}{P_i + \sum_{j=1(j \neq i)}^{n-1} P_{ij}}. \quad (12)$$

To determine the probabilities P_{iy} and $P_{ijy}, (i = \overline{1, n}; j = \overline{1, n-1})$ we should consider subsets $E_i, (i = \overline{1, n})$ as independent subsets. For the comfort the set of numbers j is marked as $J, (j \in J)$. Let's introduce numbers m , also belonging to this subset ($m \in J$). With regard of the new numbers of dependence to determine P_{iy} and P_{ijy} are:

$$P_{iy} = \frac{1}{1 + \sum_{j=1(j \neq i)}^{n-1} \rho_j} , \quad (13)$$

$$P_{ijy} = \frac{\rho_j}{1 + \sum_{m=1(m \neq i)}^{n-1} \rho_m} , \quad (14)$$

where $\rho_j = \frac{\lambda_j}{\mu_j}$, $\rho_m = \frac{\lambda_m}{\mu_m}$ - the imported intensities of streams.

Replacing (13) in (10) we will receive:

$$\mu_{i0} = (1 + \sum_{j=1(j \neq i)}^{n-1} \rho_j)^{-1} \mu_i . \quad (15)$$

All intensities on graph (fig.5) are known, and the probability of states $P_0, P_{1\Sigma}, \dots, P_{i\Sigma}, \dots, P_{n\Sigma}$ are defined according to well-known dependences [3]:

$$P_0 = \left[1 + \sum_{i=1}^n \rho_i (1 + \sum_{j=1(j \neq i)}^{n-1} \rho_j) \right]^{-1} , \quad (16)$$

$$P_{i\Sigma} = \left[1 + \sum_{i=1}^n \rho_i (1 + \sum_{j=1(j \neq i)}^{n-1} \rho_j) \right]^{-1} \rho_i (1 + \sum_{j=1(j \neq i)}^{n-1} \rho_j) . \quad (17)$$

After calculations according to dependence (16), (17) the graph probabilities conditions, introduced on fig. 5, can determine the probability of states $S_i (i = 1, \bar{n})$ and $S_{ij} (i = 1, \bar{n}; j = 1, \overline{n-1})$ of graph, introduced on fig. 3. According to (11), (12) and (13),(14):

$$P_i = P_{iy} P_{i\Sigma} = (1 + \sum_{j=1(j \neq i)}^{n-1} \rho_j)^{-1} P_{i\Sigma} , \quad (18)$$

$$P_{ij} = P_{ijy} P_{i\Sigma} = \frac{\rho_j}{1 + \sum_{m=1(m \neq i)}^{n-1} \rho_m} P_{i\Sigma} . \quad (19)$$

After replacing (17) in (18) and (19) :

$$P_i = \left[1 + \sum_{i=1}^n \rho_i (1 + \sum_{j=1(j \neq i)}^{n-1} \rho_j) \right]^{-1} \rho_i , \quad (20)$$

$$P_{ij} = \left[1 + \sum_{i=1}^n \rho_i \left(1 + \sum_{j=1(j \neq i)}^{n-1} \rho_j \right) \right]^{-1} \rho_i \rho_j. \quad (21)$$

The whole initial structure of the flexible synchronous line (FSPL) from multirole machines SN, including reserve working place (RTC), is replaced through one simplest equivalent element for which the intensities of the failures streams λ_{Σ} and the restoration of efficiency μ_{Σ} are known. Element with two states is considered as the simplest: the emergency and working state. Graph of conditions of such element is shown on fig.6.

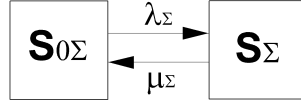


Fig. 6. Graph of FSPL conditions, referred to the simplest element

States on the graph: $S_{0\Sigma}$ - able to the work; S_{Σ} - broken (unable to work). We will introduce two new subsets of states for graph on fig. 4: U - able to work, encircled with dashed line, and V - incapable to work:

$$U = \{S_0, S_1, \dots, S_i, \dots, S_n\}$$

$$V = \{S_{1,1}, \dots, S_{1,j}, \dots, S_{1,n-1}, \dots, S_{i,1}, \dots, S_{ij}, \dots, S_{i,n-1}, \dots, S_{n,1}, \dots, S_{n,j}, \dots, S_{n,n-1}\}$$

The subset U answers state $S_{0\Sigma}$ introduced on fig. 6, and the subset V - state S_{Σ} . The probability of system being in states $S_{0\Sigma}$ and S_{Σ} is equal:

$$P_{0\Sigma} = P_0 + \sum_{i=1}^n P_i, \quad (22)$$

$$P_{\Sigma} = 1 - P_{0\Sigma} = \sum_{i=1}^n \sum_{j=1(j \neq i)}^{n-1} P_{ij}. \quad (23)$$

Intensities λ_{Σ} and μ_{Σ} for graph introduced on fig. 6 are equal to:

$$\lambda_{\Sigma} = \sum_{i=1}^n \left(\frac{P_i}{P_{0\Sigma}} \sum_{j=1(j \neq i)}^{n-1} \lambda_j \right) = \sum_{i=1}^n \left(\frac{P_i}{P_0 + \sum_{i=1}^n P_i} \sum_{j=1(j \neq i)}^{n-1} \lambda_j \right), \quad (24)$$

$$\mu_{\Sigma} = \sum_{i=1}^n \sum_{j=1(j \neq i)}^{n-1} \frac{P_{ij}}{P_{\Sigma}} \mu_j = \sum_{i=1}^n \sum_{j=1(j \neq i)}^{n-1} \frac{P_{ij}}{\sum_{i=1}^n \sum_{j=1(j \neq i)}^{n-1} P_{ij}} \mu_j. \quad (25)$$

4. THE PROGRAM OF DEFINING THE FSPL RELIABILITY AND EFFICIENCY

The program of defining the parameters of functioning synchronous FSPL was written in the mathematical software for analytic calculations – Maple. This environment is strong computer tool, able the solve complexed mathematical tasks. It contains tools related to many mathematical fields (algebra, discrete mathematics, differential and integral mathematics, numerical and different methods) and also allows graphical representation, connections to external modules and programming tools. Block pattern of calculating parameters of functioning the synchronous line was shown on fig. 7.

The components of the program:

1. Block pattern of the entrance data.
2. The block of the calculation required parameters functions the synchronous line with (without) the reserve place.
3. Block of formating results of the experiment and output for these results.

Input data to the execution of research:

1. the maximum amount of cells in the line N ;
2. the intensity of the stream of damages λ_i and restoring the working ability μ_i of every unit ($i = \overline{1, N}$);
- 3. the average time of service for every production individual cell t_i ($i = \overline{1, N}$);**
4. step of calculations Δn (the total number equal to difference between the values of two of the current quantity of cells in line n of neighbouring cycles).

The block of calculations contains following operations:

1. Defining intensities of streams $\rho_i = \frac{\lambda_i}{\mu_i}$, $i = \overline{1, N}$
2. The qualification of intensity μ_{i0} according to dependence (15).
3. Calculation of the probability P_0 according to dependence (16).
4. The calculation of the probability $P_{i \Sigma}$, P_i , P_{ij} according to dependence (17), (18), (21) suitably.
5. Qualification rate of the readiness of the line $K_T = P_{0\Sigma}$ according to dependence (22).

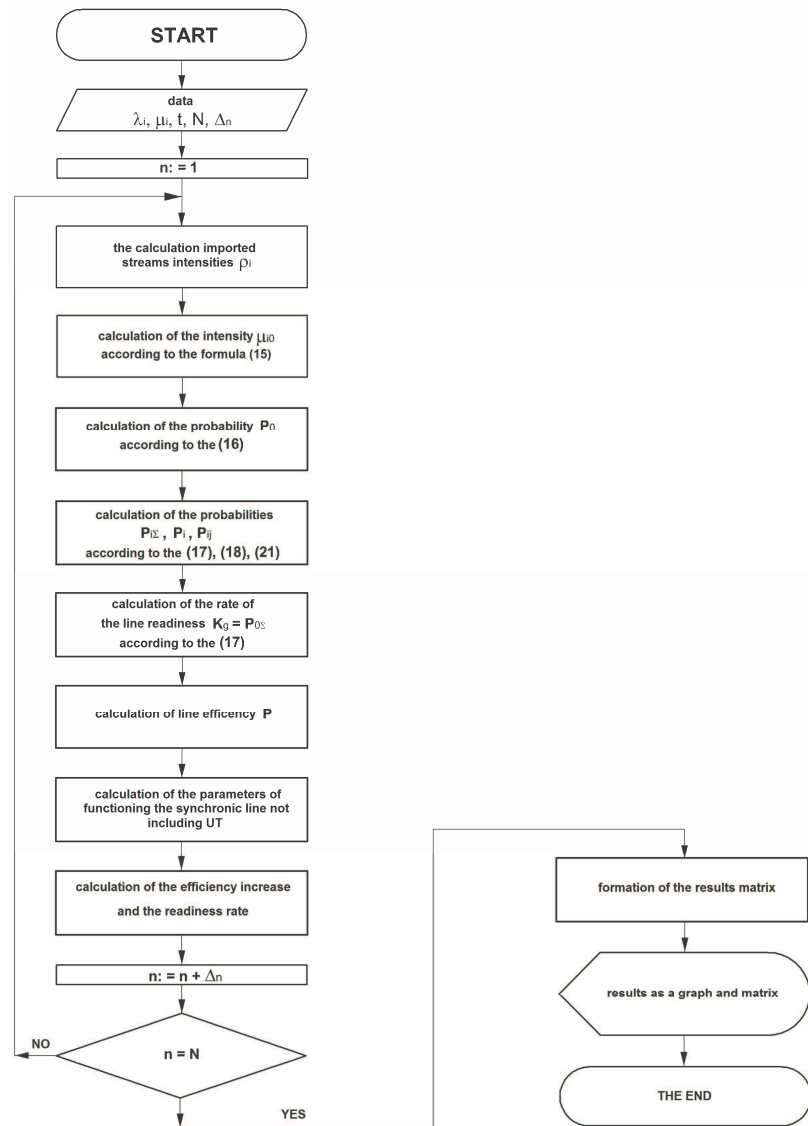


Fig.7. The block pattern of calculations of the functioning parameters of the synchronous line

6. Calculation of the efficiency of the line: $II = \frac{1}{t_{\max}} K_{\Gamma}$,

where t_{\max} - maximal time among the average times of the service for every of the production cell

7. Defining the parameters of functioning the synchronous line not including the reserve place:

- the rate of the readiness of the line $K'_r = \frac{1}{1 + \sum_i \rho_i}$,

- the efficiency of the line $\Pi' = \frac{1}{t_{\max}} K'_r$.

8. Calculation of current values

- increase of the coefficient of the readiness of the line
as the absolute value $\Delta K_r = K_r - K'_r$

in percentages $\delta K_r = \frac{\Delta K_r}{\max\{K_r, K'_r\}} 100\%$

- increase of the efficiency of the line
as the absolute value $\Delta \Pi = \Pi - \Pi'$

in percentages $\delta \Pi = \frac{\Delta \Pi}{\max\{\Pi, \Pi'\}} 100\%$

These calculation are taken cyclically until the condition $n = N$ is not met. After that the programme works out the results of experiment (the increase of efficiency) and presents these results on the screen as the matrix and chart. The charts of increase of FSPL efficiency after changing parameters λ and μ were introduced on fig. 8, 9, 10.

5. THE RESULTS OF CALCULATIONS

5.1 Line With The Maximum Number TC Equal To 10

The parameters of reliability and service of all TC are equal

$\lambda = 0,2 (\mu-1)$

$\mu = 5 (\mu-1)$

$t = 0,1 (\mu)$

THE MATRIX OF THE EFFICIENCY INCREASE

3.84615384615385
7.13305898491084
9.94897959183673
12.3662306777646
14.4444444444444
16.2330905306972
17.7734375000000
19.1000918273646
20.2422145328720
21.2244897959184

MAXIMUM INCREASE

efficiency increase

21.224489795918

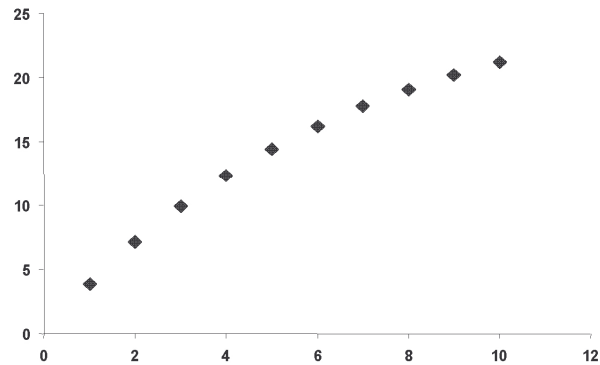


Fig. 8. The graph of FSPL

5.2 The Line With The Maximum Number Of TC Equal 10

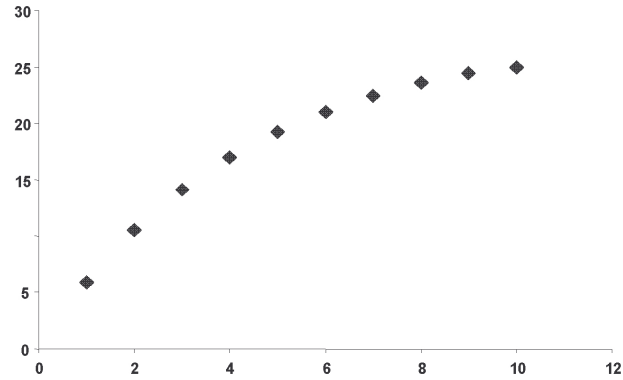
The parameters of reliability and service of all TC:

$$\lambda = 0,25 (\mu-1)$$

$$\mu = 4 (\mu-1) \qquad t = 0,1 (\mu)$$

THE MATRIX OF THE EFFICIENCY INCREASE

```
5.88235294117647
10.4938271604938
14.1274238227147
17.0000000000000
19.2743764172336
21.0743801652893
22.4952741020794
23.6111111111111
24.4800000000000
25.1479289940828
```



MAXIMAL INCREASE
25.1479289940828

Fig. 9. The graph of FSPL efficiency increase

5.3 The Line With The Maximum Number Of TC Equal 10

The parameters of reliability and service of all TC:

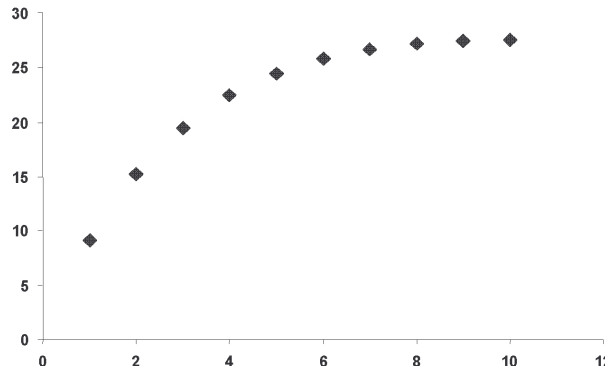
$$\lambda = 0,3 (\mu-1)$$

$$\mu = 3 (\mu-1)$$

$$t = 0,1 (\mu-1)$$

THE MATRIX OF THE EFFICIENCY INCREASE

```
9.09090909090909
15.2777777777778
19.5266272189349
22.4489795918367
24.4444444444444
25.7812500000000
26.6435986159170
27.1604938271605
27.4238227146814
27.5000000000000
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MAXIMAL INCREASE
27.5000000000000

Fig. 10. The graph of FSPL efficiency increase

6. CONCLUSIONS

The report shows the place of multi–role CNC machines for holes processing in frame type parts in real conditions. Modelling methodology of reliability and the efficiency of flexible productive lines, including reserved machine able to replace any machine in line was introduced. The results of modelling prove that at the suitable choice of technological devices, efficiency and reliability of FPS increases.

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